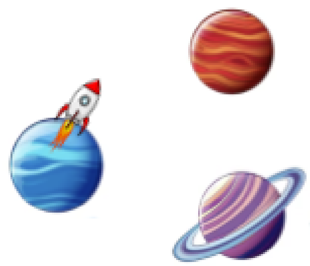
1. (7 points) A rocket randomly hops between 3 planets. On each turn, the rocket hops to one of the other 2 planets with equal probability. After 2 turns, what is the probability that the rocket has returned to the planet where it started? Write your answer as a fraction.



2. (7 points) There are 3,000 watermelons. 976 watermelons go bad. Bob buys all of the good watermelons. If Bob evenly distributes all of those watermelons among 23 people, how many watermelons will each person have?

3. (7 points) What is the smallest area bounded by the unit circle centered at $(0, 0)$ and the lines y=0 and y=x?

---

4. (8 points) For positive real numbers x, y, z, let $x + y + z = 8$. If the largest value of xyz can be expressed as $\frac{a}{b}$, find $a + b$.

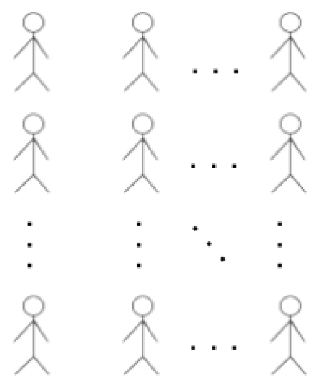
5. (8 points) A palindrome is a number that is read the same forward or backward. For example, 9009 or 727 would be palindromes, while 9019 or 722 would not. How many five-digit palindromes are divisible by 3?

6. (8 points) Saul travels to three soccer fields to sell his soccer balls. The total number of balls he sells each time is half of his stock plus one half. At the end of all this, he ends up with 0 soccer balls and he didn't cut any soccer balls in half. How many soccer balls did he start with?

7. (9 points) Find the total number of perfect squares between 1 and 2024.

8. (9 points) Villages A and B are 2.8 kilometers apart. John starts walking from Village A to Village B, and after 5 minutes, Bessie starts cycling from Village B to Village A. They meet 10 minutes after Bessie starts. Bessie cycles 130 meters more per minute than John walks. What is John's walking speed (in meters per minute)?

9. (9 points) If a group of students tries to stand in a filled square formation (see diagram below), there are 10 extra students. If they try to stand in a filled square formation with each side extended by one person, they are 15 people short. How many students are there?



---

10. (10 points) Given the following system of equations, what is the value of $x + y + z$?

$$\begin{aligned} 2x + 3y - z &= 10 \\ -x + 4y + 2z &= -2 \\ x - y + z &= 1 \end{aligned}$$

11. (10 points) A bottle contains 1000 grams of a 15% alcohol solution. When 100 grams of Solution A and 400 grams of Solution B are poured into the bottle, the alcohol concentration in the bottle becomes 14%. Given that the alcohol concentration of Solution A is twice that of Solution B, what is the alcohol concentration of Solution A?

12. (10 points) Given that the product of 2016 and a positive integer z is a perfect square, what is the minimum value of z?

13. (11 points) Let $a \wedge b \equiv ab \pmod{a+b}$ and $a \circ b \equiv a^2b^2 \pmod{2ab}$ where $x \mod y$ is the remainder of x when divided by y. What is $(3 \circ (3 \wedge 4)) \wedge 2$?

14. (11 points) Triangle $\triangle ABC$ has side lengths $AB = 8, BC = 17, CA = 15$. Let O be the incircle of $\triangle ABC$ and intersect AB, BC, and CA at X, Y, and Z, respectively. What is $\angle XYZ$?

15. (11 points) Find the number of ordered triples (x, y, z), where x, y, z are integers, such that:

$$x^2 + y^2 + z^2 = 4x + 4y + 4z + 13$$

---

16. (12 points) The value of $\frac{2}{1!} - \frac{3}{2!} + \frac{4}{3!} - \frac{5}{4!} + ... - \frac{2025}{2024!}$ can be written as $a - \frac{1}{b!}$ where a and b are positive integers. Find $a + b$.

17. (12 points) In a square of area 99, two points are randomly selected on a random side, while a third point is randomly selected on a different side. These three points form a triangle. Find the area of the region covered by the centroids of all such triangles.

18. (12 points) In a internally reflective circle with diameter $\overline{AB}$, a laser is shot from A to A' on the circle, and returns to point A on the 30th reflection. Find the sum of all possible values of $\angle BAA'$ in degrees.

---

19. (13 points) Suppose two six-sided dice are rolled, and a coin is tossed the same number of times as the product of the two numbers. Find the expected value of heads to appear on a given roll.

20. (13 points) Find the least positive integer k such that $20242012 \times 20242016 \times 20242020 \times 20242024 + k$ is a perfect square.

21. (13 points) In a regular tetrahedron $ABCD$ with volume 1024, denote the incenter as X. Find the volume of the set of all points in $ABCD$ that is closer to X than any vertex $A, B, C,$ or D.