Set Alpha Solutions:

1. **2.5 children.** Solution:
   $0.1 \cdot 0 + 0.2 \cdot 1 + 0.1 \cdot 2 + 0.3 \cdot 3 + 0.3 \cdot 4 = 2.5\ children$

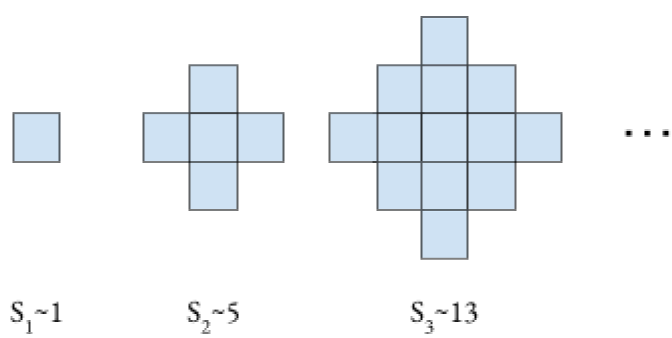
2. **6.25 million.** Solution:
   Because the growth rate from one population to the next is $\frac{children}{parents} = \frac{2.5}{2}$, the reproducing population two generations down the line will be $4 \cdot (\frac{2.5}{2})^2 = 6.25\ million$

3. **240**. Solution:
   Treat the pair of needy students as one student. 5 students could be seated in 5! = 120 ways. However, we must multiply this by 2 to account for Nino and Elene swapping places. Thus, we get **240** possible seating arrangements.

4. **120**. Solution:
   $240 = 2^4 \cdot 3 \cdot 5$
   $2^3 \cdot 3^2 \cdot 5 = 360$
   $360 - 240 = 120$

Set Beta Solutions:

5. **61**. Solution:
   1+4+8+12+16+20=**61** squares



6. **7**. Solution:
   $\frac{\sqrt{3}}{4} \rightarrow 7$

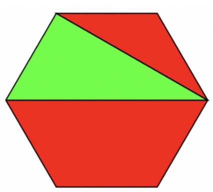
7. **5**. Solution:

# day	1	2	3	4	5
# in jar	7+2=9	9+1=10→5	5+1=6→3	3+1=4→2	2→1
# owed	2	2+1=3	3+1=4	4+1=5	**5**

8. **100**. Solution:
   Form the rectangle with two rows from the 5 rows as the horizontal sides and two columns from the 5 columns as the vertical sides.
   (5C2)*(5C2)=**100**

Set Gamma Solutions:

9. **2/3**. The area of a regular hexagon can be expressed as $\frac{3\sqrt{3}}{2}r^2$, where r is the radius of the circle circumscribing the hexagon. This is derived by splitting a regular hexagon into 6 congruent equilateral triangles.
   To calculate the probability that a dart lands on the red area, we must find the ratio $\frac{area\ of\ red\ part}{total\ area}$. However, it is easier to calculate the green area, so we can rewrite the ratio as $\frac{total\ area - area\ of\ green\ part}{total\ area} = 1 - \frac{area\ of\ green\ part}{total\ area}$.

   The green triangle is a right triangle because its hypotenuse lies on the diameter of the circle circumscribing it. Furthermore, it is a 30-60-90 triangle, thus we know that the longer leg is $\sqrt{3}$ times the shorter one.
   The regular hexagon has the property that its side has the same length as its radius. Hence, the legs of the green triangle can be rewritten as r and r$\sqrt{3}$. The area of the green triangle is then

$\frac{r \cdot r\sqrt{3}}{2} = \frac{r^2\sqrt{3}}{2}$. Substituting this expression into our original equation yields

$$\begin{aligned} &1 - \frac{\text{area of green part}}{\text{total area}} \\ &= 1 - \frac{\frac{r^2\sqrt{3}}{2}}{\frac{3r^2\sqrt{3}}{2}} \\ &= 1 - \frac{1}{3} \\ &= \boxed{\frac{2}{3}}. \end{aligned}$$

10. **42**. Solution for Problem 10

The area of the left triangle is $\frac{12 \cdot 16}{2} = 96$. The area of the other triangle is expressed as $\frac{a \cdot b}{2}$. We are given the equation

$$96 = \left(\frac{2}{3}\right)^2 \frac{a \cdot b}{2},$$

which can be simplified to

$$\begin{aligned} 96 &= \left(\frac{2}{3}\right)^2 \frac{a \cdot b}{2} \\ 96 &= \left(\frac{2}{9}\right) a \cdot b \\ ab &= 432. \end{aligned}$$

By Power of a Point Theorem, we have the equation

$$12a = 16b.$$

(Alternatively, the Pythagorean theorem is also possible, as they are both right triangles.) Solving this system of equations, we find that $a = 24$ and $b = 18$. Thus, $a + b = \boxed{42}$.

11. **5**. Solution for Problem 11

For each of the bases in this sum, there is a repeating pattern in the units digit when they are raised to a power. The table below shows this pattern in the units digit.

Exponent	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	0
2	1	4	9	6	5	6	9	4	1	0
3	1	8	7	4	5	6	3	2	9	0
4	1	6	1	6	5	6	1	6	1	0

$42 \equiv 2 \pmod 4$, thus we look at the second row in our table. Adding up the digits gives $1+4+9+6+5+6+9+4+1+0=45$. We are looking for units digit of this sum, so our final answer is $\boxed{5}$.

12. **3157**. Solution for Problem 12

The expression in parentheses simplifies nicely by multiplying both denominators with theur conjugates:

$$\begin{aligned}
&\left(\frac{1-2i}{3+4i}-\frac{2+i}{5i}\right)^5 \\
&=\left(\frac{1-2i}{3+4i}\cdot\frac{3-4i}{3-4i}-\frac{2+i}{5i}\cdot\frac{-i}{-i}\right)^5 \\
&=\left(\frac{3-4i-6i+8i^2}{9-16i^2}-\frac{-2i-i^2}{-5i^2}\right)^5 \\
&=\left(\frac{3-10i-8}{9+16}-\frac{-2i+1}{5}\right)^5 \\
&=\left(\frac{-5-10i}{25}+\frac{2}{5}i-\frac{1}{5}\right)^5 \\
&=\left(-\frac{1}{5}-\frac{2}{5}i+\frac{2}{5}i-\frac{1}{5}\right)^5 \\
&=\left(-\frac{2}{5}\right)^5 \\
&=\boxed{-\frac{32}{3125}}.
\end{aligned}$$

32 + 3125 = **3157**

Set Delta Solutions:

13. **2024**. Solution:

A nice number theory trick can be used here. It is easier to find n+1 rather than n. The only possible value for n+1 is 3^4*5^2 as it must be a perfect square as well as be divisible by 3 and 5.

Therefore as n+1 is 2025, n is 2024.

Answer: n=2024

14. **6**. Solution:
    Again, this question seems to be a difficult counting question at first with 25 different possibilities, but a close examination gives a simple answer. First put Daniel on any ship. Then, the possibility Eric is on the same ship is ⅕. Therefore, a+b=6.

Answer: o=6

15. **28**. Solution:
    This question contains one of my favorite principles in counting: The Pigeonhole Principle.
    By pigeonhole, there are only 8 different weeks when two soldiers were born.
    To create a group of 4 soldiers with only two unique birth weeks, one can choose 2 groups out of the 8.
    By combination counting, 8 C 2 = 28

Answer: p=28

16. **26**. Solution:
    This question follows the world-famous circle permutation.
    Given that each person must sit across from their best friend, it is easier to think how to seat p/2 people as placing the other p/2 people comes with only 1 case.
    Given that p/2 is 14 and the circle permutation for n is n!/n, and there are 2^14 ways to alternate each pair.
    However, note that each pair is double counted, so we divide by 2.
    The answer is 13!*2^13.

Answer: 26