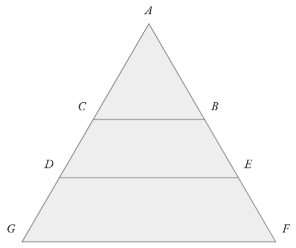
DMC 2023 Team General Round(240 points)
April 30th, 2023 Name: _____

1. (18 points) Arvin ordered a pizza with 12 slices. Arvin ate one slice, then took another slice and shared it equally with his friend Jamal. What percentage of the pizza did Arvin eat?

2. (21 points) Arvin loves swimming laps in the pool. When he first started, he completed 10 laps in 25 minutes. After a year of training, he can now finish 12 laps in 24 minutes. By how many minutes has Arvin improved his lap time?

3. (24 points) A point inside square ABCD has distances $5\sqrt{2}$, $5\sqrt{10}$, $15\sqrt{2}$, and $5\sqrt{10}$ from the vertices. Find the area of ABCD.

4. (27 points) An equilateral triangle is split into 3 sections with the same area by cuts parallel to one of the triangle's sides, as shown. What is the ratio between the heights of upper triangle ABC and lower trapezoid DEFG? (image not drawn to scale.)



5. (33 points) Calculate the sum $\binom{100}{0} + \binom{100}{2} + \binom{100}{4} + \binom{100}{6} + ... + \binom{100}{100}$.

6. (36 points) Let a, b, c, d, e, f be positive integers which satisfy the following: $abcdef = a + b + c + d + e + f$. Find the maximum possible value of $max\{a, b, c, d, e, f\}$ (the operator $max\{\}$ takes the largest number in the set, i.e. $max\{1, 3, 4, 2\} = 4$).

7. (39 points) Let P be the product of the roots of $z^6 + z^4 + z^3 + z^2 + 1 = 0$ that have a positive imaginary part, and suppose that $P = r(\cos\theta^\circ + i\sin\theta^\circ)$, where $0 < r$ and $0 \leq \theta^\circ < 360$. Find θ.

8. (42 points) In how many ways can you tile a 1×10 grid with 1×1 and 1×2 blocks?