DMC 2023 Individual Theme Round(80 points)
April 30th, 2023 Name: _____

1. (6 points) Matt Damon is trapped on Mars. He currently has 50 potatoes in storage, but will eat 10 potatoes a day to survive. However, Matt has a farm that produces 5 potatoes every morning. For how many days will the stored potatoes last?

2. (7 points) On an exoplanet of radius 2500km, a space shuttle flies 1 km above the ground along the equator. Find the least integer speed for the space shuttle in km/h such that the space shuttle can travel 1 whole circle around the world in 3π hours?

3. (8 points) On the Meandering Earth, a grid of solar panels is constructed by CloseAI's quantum intelligence 550M, consisting of 576 equilateral triangles with side length 1m, each with a circular panel inscribed. What is the area of the unused space on the grid in m^2?

4. (9 points) Over the course of the next 7 years, your spaceship has to travel exactly 0.35 of a light year. Since you don't want to waste precious fusion fuel on decelerating or maintaining speed, every year you have to travel more distance than the previous year and it must be a whole multiple of 0.01 light years. In how many ways can you plan the annual distance travelled for the next 7 years?

5. (11 points) Neptune has rings, located in a 2-dimensional disk 5.3×10^4 to 6.3×10^4 km away from the planet. The density of particles per square meter is inversely proportional to their distance from Neptune's center. Scientists have identified two particles in the ring region at random, what is the probability that the difference between the radii of these two particles' orbits is less than 4,000 km?

6. (12 points) An asteroid is hurtling towards your spaceship with an acceleration of 10 km/s^2 and initial velocity of 20 km/s from a distance of 180km. However, you have an asteroid interception satellite, which rotates around your spaceship at a speed of 10π km/s with an initial position right between you and the asteroid. Find the least possible positive integer in km to which you can set the orbit of your interception satellite so that it would intercept the asteroid.

7. (13 points) In 2403, the Earth is on the brink of destruction from an dual vector foil attack from an alien civilization which collapses the entire solar system into 2 dimensions. However, there is one lightspeed spaceship which could help people escape the fate of being splattered onto a 2D painting. Members of the human population are chosen by lottery ticket to board the spaceship. In Eric's district, the lottery requires him to pick six different integers from 1 through 25, inclusive. He chooses his numbers so that the sum of the base-six logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property— the sum of the base-six logarithms is an integer. What is the probability that Eric holds the winning ticket?

8. (14 points) You are an advanced alien civilization called the Trisolarians who have just received friendly communications from an innocent planet 4 light years away called Earth. Orbiting in an extremely chaotic three-body system, your planet is living hell and your civilization desperately wants to find another inhabitable planet, so you decide to colonize Earth. You determine that your civilization's technology level is 1 level above Earth. However, every 100 years, the probability of a civilization increasing its technology level by 1 through a technological boom is 20%, and when they fail to increase their technology level this century, their chances for increasing it the next century will be doubled (if it exceeds 100% it is guaranteed). Your spaceships can achieve a velocity of one one-hundredth the speed of light. Assume that whenever the technology level of the Trisolarians advance, the advancement is synced with the technology level of the space fleet due to quantum entanglement communication technologies. What is the probability (to 2 significant figures) that when you reach Earth, you would have a higher technological level than Earth so as to ensure its utter destruction?