DMC 2023 Individual Relay Round(80 points)
April 30th, 2023 Name: ______________________________

Set Alpha

1. (1 point) Mr. P splits 10 pieces of candy among three children. Assuming each child gets at least one piece of candy, find α_1, the maximum number of pieces of candy a child could get.

2. (3 points) On the second day, one child got α_1 pieces of candy but could only eat 3. Find α_2, the number of ways he could split his remaining pieces of candy to his two friends (giving one friend all his remaining candy and leaving none for his other friend counts as a valid way of splitting).

3. (5 points) Find α_3, the area of the circle in which an equilateral triangle of side length α_2 can be inscribed.

4. (7 points) If $k = 3p^2 + 2p + 1$ and $f(x) = k\pi x^2 + \frac{\alpha_3}{3} + 2\pi$, find α_4, the sum of all possible real values of p which make $f(x)$ have a repeated root?

Set Beta

1. (2 points) Bob finds the prime factorization of 12!. What is β_1, the sum of exponents of Bob's prime factorization?

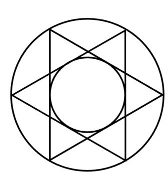
2. (4 points) Bob went to the supermarket and bought $\beta_1 - 10$ identical bananas for his 4 friends, Boba, Bonnie, Billy, and Bobert. If Boba insists she gets at least two bananas and the rest of Bob's friends don't care how many bananas they get, in how many ways β_2 can Bob distribute the bananas to his 4 friends?

3. (6 points) At Bob's school, β_2 kids like banana splits, 63 kids like banana pancakes, and 109 kids like banana pudding. Each student can choose to like none, one, two, or all three deserts. Only Billy and Bobert, 2 kids, don't like any of the deserts. If 194 kids attend Bob's school and 80 kids like at least two of the deserts, how many students, β_3, like all three options?

4. (8 points) Bob is standing at the bottom left corner of a rectangular grid with two consecutive integers as side lengths and with area β_3. Bob moves only up and right one unit at a time. Find β_4, the number of ways Bob can move to the top right corner.

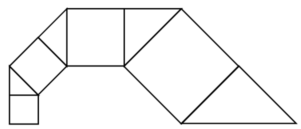
Set Gamma

1. (2 points) Given that all the shapes in this diagram are regular, find γ_1, the ratio of the big circle's area to the small circle's area.



2. (4 points) James saved \$1000 in the bank. The bank multiplied James's savings by $\sqrt{\sqrt{2}}$ every single year and, after 64 years, James now has \$65 536 000. James calculates that if the bank had multiplied his savings by γ_1 every other year instead by $\sqrt{\sqrt{2}}$ every single year, it would have taken γ_2 years for his initial \$1000 to reach \$65 536 000. Find γ_2.

3. (6 points) A sequence of shapes is created as such with squares and right isosceles triangles, where A_n denotes the shape containing n squares and n triangles (therefore, this diagram shows A_4). If the smallest square always has a side length of 1, find γ_3, the area of A_{γ_2}.



4. (8 points) The following pieces of information are known:
   - In the town of Deerfield, all women have a haircut exactly every γ_4 days.
   - The hairdressers give haircuts to exactly 2^{12} men and exactly 2^{10} women per day.
   - There are $(\gamma_3 + \frac{3}{2})$ men in Deerfield
   - Deerfield's male to female ratio is 3:2

   What must be the value of γ_4?

Set Delta

1. (3 points) Find δ_1, the sum of all solutions to the following: $10^x + 11^x + 12^x = 13^x + 14^x$.

2. (5 points) Find δ_2, the sum of all real number(s) x that satisfy the following equation:

$$\frac{8^x + 27^x}{12^x + 18^x} = \frac{7}{3\delta_1}$$

3. (7 points) Andrew and Benjamin plan to meet up for lunch. They decide that they will each independently show up at a random time between 11am and 1pm. They will each wait for $15+\delta_2$ minutes; if the other shows up in that time period, they will enjoy lunch together. Otherwise, they will just leave. Today, Andrew showed up, waited for $15+\delta_2$ minutes, and left because Benjamin had not shown up. The probability that they would have had lunch together if Andrew had been willing to wait $15+\delta_2$ more minutes is a fraction of the form $\frac{p}{q}$. Find, $\delta_3 = p + q$.

4. (9 points) We form a word using only D's, M's and C's. Suppose we can never have an M next to a C. Find δ_4, the number of x-letter strings that can be formed, where x is sum of the digits of $2\delta_3$.