DMC 2025 Individual Theme Round (300 points)
April 26th, 2025

1. (20 points) In the Table Tennis Men's Singles semifinal round, bronze medalist Felix Lebrun won 26 points across 4 games, and the points he won each game forms an arithmetic sequence increasing by 1. What is the most number of points he scored in a game?

2. (25 points) At the Opening Ceremony, the 12 countries from South America could have lined up in $12! = 479,001,X00$ ways to travel down the Seine River, where X is a single digit. What is X?

3. (30 points) SoFi Stadium—the 2028 Olympics stadium in Los Angeles that will be used for opening and closing ceremonies—can seat 432,000 spectators. The number of seats in each subsequent row increases by a constant amount, with the first row seating 1,000 people and the last row seating 35,000. Including the first and last rows, how many rows of seats are there?

4. (35 points) In Paris, a street vendor sells croissant sandwiches for either 7 or 11 Euros. What is the largest integer quantity of Euros that can't be the exact payment for a purchase of croissants?

5. (40 points) Two boxers are fighting each other in a square ring of side length one, where each corner is labeled A, B, C, and D. Before the match starts, the strong American Boxer starts at the midpoint M between A and D and then walks to a point E along $\overline{BM}$ such that $\overline{AE} \perp \overline{BM}$. The clueless British boxer starts at point B before moving to a point F along $\overline{BM}$ such that $\overline{CF} \perp \overline{BM}$. The British boxer didn't hear the bell go off and remained situated at point F. What distance will the American boxer have to travel to punch the British boxer?

6. (45 points) Every hour, a fraction of the Olympic Phrygian memorabilia, based on the French red hat mascots of Paris 2024, is sold. During the nth hour, $\frac{2}{(4n^2-1)}$ of the total memorabilia is sold. After 2025 hours, the fraction of the total memorabilia that has been sold can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

7. (50 points) In a round-robin style event, 6 teams compete. Each team plays exactly 2 other teams, randomly assigned. There are no draws. Find the number of possible outcomes.

8. (55 points) A Skeet shooter must aim his shotgun at a target with radius 6 centered around the origin. The region that the shotgun's pellets will scatter (hit every point) can be be represented by the inequality,

$$k|x| + (17-k)|y| \leq 78$$

where k is a constant. The range of k for which the pellets hit every point on the target is $[m, n]$. Find mn.

DMC 2025 Individual Relay Round (300 points)
April 26th, 2025

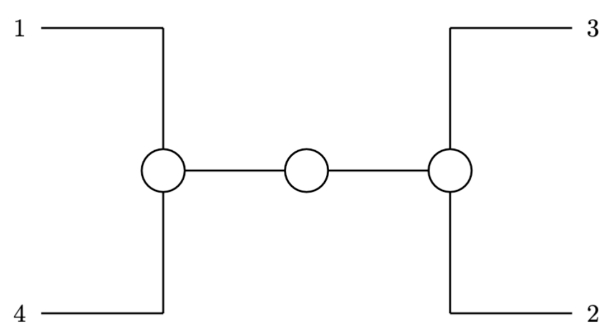
Set Alpha (60 Points)

1. (12 points) Amanda and Andy love candies. On a given day, Amanda has 22 more candies than Andy. However, after reading Marx's Communist Manifesto, Amanda and Andy, inspired, abandon the principles of capitalism in pursuit of economic equality and decide to redistribute their candies so that both have the same number of candies. How many candies (α_1) does Amanda have to give to Andy so that they could achieve this noble goal?

2. (14 points) Let α_2 be a positive integer such that $\alpha_1 = \alpha_2 + 2^{\alpha_2}$. Find α_2.

3. (16 points) Two lines, l and m, run parallel to each other. Two other lines, line a and b, both intersect l and m once so that they intersect somewhere between l and m at point I. The four lines form two triangles in the middle. The smaller triangle, with a segment of l as one of its sides, has sides length 5 and 2 on the non-l sides, and the larger triangle has side length α_2 on one of the non-m sides. Let the largest possible length of the other non-m side be n. Find $\alpha_3 = \lfloor n \rfloor$.

4. (18 points) In a March Madness tournament, the Final Four plays eliminating matches for two rounds to produce a winner. All teams are seated from seed numbers 1 to 4, from best to worst, and their games are arranged as the diagram suggested (circles represent matches). The probability for a team to win their game is $\frac{m}{m+n}$, where n is their seed number, and m is the seed number of the opposing team. Find α_4, the probability that team $(\alpha_3 - 5)$ will win the tournament.



Set Beta (70 Points)

5. (15 points) Muhammad is preparing for the school dance, and he is having a hard time choosing what to wear. He can choose from 5 white shirts, 1 long khaki pants, 2 blazers, 3 ties, and 1 belt. Find β_1, the number of combinations of fits that he can wear, given he must choose exactly one of each item.

6. (17 points) Let the sum of the area of two circles A and B be β_1. The area of B is the cube of the area of A. Find β_2, the ratio of the area of B to the area of A.

7. (18 points) Compute the remainder β_3 when β_2^{2028} is divided by 2027.

8. (20 points) Find the unique positive integer β_4 such that:

$$(1^3 + 2^3 + \cdots + \beta_4^3) - 90(1 + 2 + \cdots + \beta_4) + 25\beta_3 = 0$$

Set Gamma (80 Points)

9. (17 points) Kevin writes five numbers on a whiteboard. They are $2329, 2930, 2765, 2839$, and 2239. He erases one of the numbers, and the sum of the remaining four numbers is a multiple of 4. Find γ_1, the sum of all the digits of the number he erased.

10. (19 points) N is a positive integer such that its base $\gamma_1 - 8$ representation is $\overline{abc}$ and its base $\gamma_1 - 5$ representation is $\overline{cba}$. Find γ_2, the base 10 representation of N.

11. (21 points) A triangle ABC has an area γ_2. Define the following points A_n on side BC as follows:

    i) Let A_1 be the midpoint of BC

    ii) Let A_i be the midpoint of BA_{i-1} for each positive integer $i \geq 2$

    Let γ_3 be the value of the infinite sum: $[AA_1A_2] + [AA_3A_4] + [AA_5A_6] + [AA_7A_8] \cdots$, where [ ] denotes area.

12. (23 points) When a projectile is launched with initial velocity v and launch angle θ, its horizontal range is $R_x = \frac{v^2 \sin(2\theta)}{2g}$, where $g = 10\frac{m}{s^2}$ is the gravitational constant of the Earth. The total flight time of the projectile can then be represented as $t = \frac{2v \sin\theta}{g}$. A projectile can have the same range R for two different launch angles, θ_1 and θ_2. Let t_1 and t_2 be the respective flight times of the projectiles launched at $\theta_1 = \gamma_3$ and θ_2, in degrees. The product of t_1 and t_2 can be represented as $\frac{k}{g}R$. Find the value of γ_4, defined as the sum of the angle value of θ_2 in degrees and the value of k.

Set Delta (90 Points)

13. (20 points) In a population of 24 axolotls, there are 12 blue and 12 red, of which exactly half are male and the other half female, respectively. Given a biologist randomly assigns 12 male-female pairs (such that each axolotl is included), $\frac{p}{q}$ is the probability that all pairs are the same color, where p and q are relatively prime positive integers. Find $\delta_1 = p + q$.

14. (22 points) Find δ_2, the number of factors of $3^3 \cdot 5^5 \cdot 7^7 \cdot \delta_1$ with the property that it leaves a remainder of 3 when divided by 4.

15. (23 points) Find δ_3, the sum of the squares of the (not necessarily distinct) roots of the polynomial:
$$32x^4 + \delta_2 x^3 + 944x^2 + \delta_2 x + 32$$

16. (25 points) John needs to put δ_3 checkers in a $\delta_3 \times \delta_3$ checkerboard. Each row and column has exactly 1 checker, and there are no checkers in the main diagonal (in positions $(1, 1)$, $(2, 2)$, etc.). Find the number of ways John can arrange the checkers.

DMC 2025 Team General Round (300 points)
April 26th, 2025

1. (20 points) Define the following operations:

   i) $a \Diamond b = a^2 - b^2$
   ii) $a \otimes b = (a+b)^2$
   iii) $a \wedge b = (2a-b)^2$

   Evaluate $(5 \Diamond 1) \otimes (3 \wedge 4)$.

2. (25 points) Let $\{a_n\}$ be an arithmetic sequence such that $a_{2025} = a_{20} + a_{25} = 1$. Find a_1.

3. (30 points) Suppose N is a positive perfect square less than 2025^2 with the property the units digit of N is also a (not necessarily positive) perfect square. Find the number of possible N.

4. (35 points) Points A, B, C, D form convex quadrilateral $ABCD$. Three sides of this quadrilateral have lengths $1, \sqrt{2}, \sqrt{3}$, in some order, and they are chords in a circle of radius 1. Find the sum of the possible values of the angle $\angle ABC$.

5. (40 points) Circles ω_1 and ω_2, with radii $r_1 \leq r_2$, centered at O_1 and O_2, respectively, are constructed so that ω_2 passes through O_1, and they intersect at points P and Q. Suppose $PQ = 10$, and the radius of ω_2 is 8. r_1^2, in simplest form, can be expressed as $a - b\sqrt{c}$. Find $a + b + c$.

6. (45 points) John follows the following process:

   i) He flips a fair two-sided coin.

   ii) If it is heads, he rolls a fair six-sided dice and records the roll. He then goes back to step i.

   iii) If it is tails, he stops the process and adds up all the values he has recorded.

   Find the expected value of the sum he calculates.

7. (50 points) Suppose $f(x)$ satisfies $f(a+b) = f(a) + f(b) + 2ab$ for all real numbers a, b. Given $f(20) = f(25)$, find $f(\frac{45}{2})$.

8. (55 points) Let $ABCD$ be a rhombus. Suppose point E is on BC such that AE bisects $\angle BAC$. Let F be a point on CD such that $\angle DAB = 2\angle AEF$. Let M and μ be the maximum and minimum values, respectively, of $\angle ABC$ such that F lies between points C and D, inclusive. Find $M - \mu$.

1. (8 points) Captain Deerfield has 5 boards, each with a single letter/digit from "DMC25". Captain Deerfield wants to arrange these boards on top of the MSB. Given that Captain Deerfield randomly orders these letters/digits, what is the probability that they spell out "DMC25"?

2. (8 points) A weird coin lands on its head $\frac{1}{3}$ of the time, tail $\frac{1}{3}$ of the time, and side $\frac{1}{3}$ of the time. What's the probability that in three flips, it only lands on the same thing?

3. (8 points) This year, 2025, the DMC is hosted on Saturday, April 26th. What is the next year that April 26th will be a Saturday?

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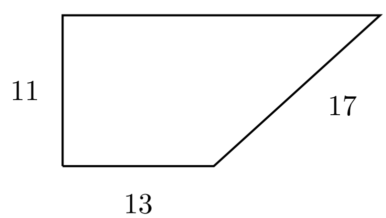
4. (10 points) The area of the trapezoid below can be represented in the form $a + b\sqrt{c}$. Find the value of $a + b + c$. The two angles on the left of the trapezoid are right angles. (Figure not drawn to scale.)



5. (10 points) What is the units digit of $2025!! = 2025 \cdot 2023 \cdot 2021 \cdot \cdots \cdot 1$?

6. (10 points) Kevin dehydrates (only removing the water) a $100g$ orange that is $\frac{4}{5}$ water until the orange is $\frac{3}{5}$ water. What fraction of the orange's initial mass is its new mass?

7. (12 points) It takes Alice 6 minutes to paint a wall, and it takes Bob 9 minutes to paint the same wall. If they work together at their constant rates, how long will it take them to paint the wall together?

8. (12 points) Consider the following system of inequalities:

$$y < 30 - 5x$$
$$\frac{y}{5} > 4$$

The inequality $x < a$ represents the set of all possible x values for all solutions (x, y) that satisfy the given system of inequalities in the xy-plane. Find a.

9. (12 points) Consider the following equation:

$$2\,(x-3)\,(x-1) = x\,(x+1)$$

One solution to the given equation can be written as $x = \frac{r+\sqrt{s}}{t}$ in simplest form. What is the value of $r + s + t$?

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10. (14 points) In triangle ABC, $AB = 13$, $AC = 15$, $BC = 14$. Points P and Q lie on BC between B and C such that $BP = CQ = 5$. Find AQ^2.

11. (14 points) Consider the function:

$$f(x) = \left|\left|x^2 + 4x\right| - 4\right| - 2$$

How many real roots does $f(x)$ have?

12. (14 points) A circle in the xy-plane has its center at $(3, 3)$. The two lines $y = \frac{1}{3}x$ and $y = 3x$ are tangent to this circle. What is the radius of this circle?

13. (16 points) Compute the sum of the positive integer factors of 105^2 that are also perfect squares.

14. (16 points) A right square pyramid $ABCDE$ has $AB = BC = CD = DA = 12$. Furthermore, the radius of the largest sphere that can be fit in $ABCDE$ is 3. Find the volume of $ABCDE$.

15. (16 points) Consider the set of all not-necessarily-simplified fractions $\frac{x}{y}$, where x and y are positive integers. How many of these have the property that if x increases by 13 and y decreases by 9, the value of the fraction increases by 25%?

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16. (19 points) A positive integer factor pair (m, n) of 2025 is selected at random. Then, two fair die are rolled, one numbered $0, 1, 2, \cdots, m$ and the other $0, 1, 2, \cdots, n$. What is the expected value of the product of the two dice?

17. (19 points) In the interior of equilateral triangle ABC, there is a point P such that the distances from AB, BC, CA are 20, 2, 5, respectively. The area of ABC, in simplest form, can be written in the form $m\sqrt{n}$. Find $m + n$.

18. (19 points) In base 10, the decimal $0.abcdabcd...$ is equivalent to the fraction $\dfrac{m}{n}$, in simplest form (where $k = \frac{k}{1}$ for integers k). How many possible values can n have?

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19. (21 points) Two positive integers n and m satisfy the following properties:

$$\gcd(m + 2025n, n) = 6$$
$$\operatorname{lcm}(m^2 n, mn^2) = 48600$$

Find mn.

20. (21 points) A frog starts on lily pad 1 of 11. When on pad n, every minute, starting right after the first minute, it has a $\dfrac{1}{n}$ chance of moving to the next pad and a $\dfrac{n-1}{n}$ of staying on the current pad. Find the expected value of minutes for the frog to reach pad 11.

21. (21 points) x, y, z are nonnegative. Find all (x, y, z) such that

$$\begin{cases} x^2 - 4x + 3y + 1024z = 16 \\ (x-2)^2 y^3 z = 1 \end{cases}$$